

About this Article

This material was included with the downloadable supplemental content accompanying the *ARRL Antenna Book*.

You may print a copy of this material for personal use. Any other use of the information requires permission from the ARRL.

Copyright/Reprint Notice

In general, all ARRL content is copyrighted. ARRL articles, pages, or documents – printed and online – are not in the public domain. Therefore, they may not be freely distributed or copied. Additionally, no part of this document may be copied, sold to third parties, or otherwise commercially exploited without the explicit prior written consent of the ARRL. You cannot post this document to a website or otherwise distribute it to other through any electronic medium.

For permission to quote or reprint material from ARRL, send a request including the issue date, a description of the material requested, and a description of where you intend to use the reprinted material to the ARRL Editorial and Production staff at: **permission@arrl.org**.

Radio Mathematics

Understanding radio and electronics beyond an intuitive or verbal level requires the use of some mathematics. None of the math is more advanced than trigonometry or advanced algebra, but if you don't use math regularly, it's quite easy to forget what you learned during your education. In fact, depending on your age and education, you may not have encountered some of these topics at all.

It is well beyond the scope of this book to be a math textbook, but this section touches some of topics that most need explanation for amateurs. For introductory-level tutorials and explanations of advanced topics, a list of on-line, no-cost tutorials is presented in the sidebar, "Online Math Resources". You can browse these tutorials whenever you need them in support of the information in this reference text.

Working With Decibels

The *decibel* (dB) is a way of expressing a ratio *logarithmically*, meaning as a power of some *base* number, such as 10 or the base for *natural logarithms*, the number $e \approx 2.71828$. A decibel, using the metric prefix "deci" (d) for one-tenth, is one-tenth of a *Bel* (B), a unit used in acoustics representing a ratio of 10, and named for the telephone's inventor, Alexander Graham Bell. As it turns out, the Bel was too large a ratio for common use and the deci-Bel, or decibel, became the standard unit.

In radio, the logarithmic ratio is used, but it compares signal strengths — power, voltage, or current. The decibel is more useful than a *linear* ratio because it can represent a wider scale of ratios. The numeric values encountered in radio span a very wide range and so the dB is more suited to discuss large ratios. For example, a typical receiver encounters signals that have powers differing by a factor of 100,000,000,000,000 (10^{14} or 100 trillion!). Expressed in dB, that range is 140, which is a lot easier to work with than the preceding number, even in scientific notation!

The formula for computing the decibel equivalent of a power ratio is

$$\text{dB} = 10 \log (\text{power ratio}) = 10 \log (P_1/P_2) \quad (1)$$

or if voltage is used

$$\text{dB} = 20 \log (\text{voltage ratio}) = 20 \log (V_1/V_2) \quad (2)$$

For equations 1 and 2 to produce equivalent results, both of the voltages must be measured across equivalent impedances. Otherwise, impedance must be accounted for according to $P = V^2 / Z$.

Positive values of dB mean the ratio is

Online Math Resources

The following Web links are a compilation of on-line resources organized by topic. Other resources are available online at www.arrl.org/tech-prep-resource-library. Look for the Math Tutorials section.

Many of the tutorials listed below are part of the *Interactive Mathematics* Web site (www.intmath.com), a free, online system of tutorials. The system begins with basic number concepts and progresses all the way through introductory calculus. The lessons referenced here are those of most use to a student of radio electronics.

Basic Numbers & Formulas

Order of Operations — www.intmath.com/Numbers/3_Order-of-operations.php
Powers, Roots, and Radicals — www.intmath.com/Numbers/4_Powers-roots-radicals.php
Introduction to Scientific Notation — www.ieer.org/classroom/scinote.html
Scientific Notation — www.intmath.com/Numbers/6_Scientific-notation.php
Ratios and Proportions — www.intmath.com/Numbers/7_Ratio-proportion.php
Geometric Formulas — www.equationsheet.com/sheets/Equations-4.html
Tables of Conversion Factors — en.wikipedia.org/wiki/Conversion_of_units

Metric System

Metric System Overview — en.wikipedia.org/wiki/Metric_system
Metric System Tutorial — see Unit 3 at www.arrl.org/radio-lab-handbook

Conversion of Units

Metric-English — www.m-w.com/mw/table/metricsy.htm
Metric-English — en.wikipedia.org/wiki/Metric_yardstick
Conversion Factors — oakroadsystems.com/math/convert.htm

Fractions

Equivalent Fractions — www.intmath.com/Factoring-fractions/5_Equivalent-fractions.php
Multiplication and Division — www.intmath.com/Factoring-fractions/6_Multiplication-division-fractions.php
Adding and Subtracting — www.intmath.com/Factoring-fractions/7_Addition-subtraction-fractions.php
Equations Involving Fractions — www.intmath.com/Factoring-fractions/8_Equations-involving-fractions.php
Basic Algebra — www.intmath.com/Basic-algebra/Basic-algebra-intro.php

Graphs

Basic Graphs — www.intmath.com/Functions-and-graphs/Functions-graphs-intro.php
Polar Coordinates — www.intmath.com/Plane-analytic-geometry/7_Polar-coordinates.php
Exponents & Radicals — www.intmath.com/Exponents-radicals/Exponent-radical.php
Exponential & Logarithmic Functions — www.intmath.com/Exponential-logarithmic-functions/Exponential-log-functionsintro.php

Trigonometry

Basic Trig Functions — www.intmath.com/Trigonometric-functions/Trig-functions-intro.php
Graphs of Trig Functions — www.intmath.com/Trigonometric-graphs/Trigo-graph-intro.php

Complex Numbers

Complex Numbers — www.intmath.com/Complex-numbers/imaginary-numbers-intro.php

that's a loss: $20 \log (0.1 / 2) = 20 \log (0.05) = -26 \text{ dB}$.

If you are comparing a measured power or voltage (P_M or V_M) to some reference power (P_{REF} or V_{REF}) the formulas are:

$$\text{dB} = 10 \log (P_M/P_{REF})$$

greater than 1 and negative values of dB indicate a ratio of less than 1. For example, if an amplifier turns a 5 W signal into a 25 W signal, that's a gain of $10 \log (25 / 5) = 10 \log (5) = 7 \text{ dB}$. On the other hand, if by adjusting a receiver's volume control the audio output signal voltage is reduced from 2 V to 0.1 V,

$$\text{dB} = 20 \log (V_M / V_{\text{REF}})$$

There are several commonly-used reference powers and voltages, such as 1 V or 1 mW. When a dB value uses one of them as the references, dB is followed with a letter. Here are the most common:

- dBV means dB with respect to 1 V ($V_{\text{REF}} = 1 \text{ V}$)
- dB μ V means dB with respect to 1 μ V ($V_{\text{REF}} = 1 \mu\text{V}$)
- dBm means dB with respect to 1 mW ($P_{\text{REF}} = 1 \text{ mW}$)

If you are given a ratio in dB and asked to calculate the power or voltage ratio, use the following formulas:

$$\text{Power ratio} = \text{antilog} (\text{dB} / 10) \quad (3)$$

$$\text{Voltage ratio} = \text{antilog} (\text{dB} / 20) \quad (4)$$

Example: A power ratio of $-9 \text{ dB} = \text{antilog} (-9 / 10) = \text{antilog} (-0.9) = \frac{1}{8} = 0.125$

Example: A voltage ratio of $32 \text{ dB} = \text{antilog} (32 / 20) = \text{antilog} (1.6) = 40$
Antilog is also written as \log^{-1} and may be labeled that way on calculators.

CONVERTING BETWEEN DB AND PERCENTAGE

You may also have to convert back and forth between decibels and percentages. Here are the required formulas:

$$\text{dB} = 10 \log (\text{percentage power} / 100\%) \quad (5)$$

$$\text{dB} = 20 \log (\text{percentage voltage} / 100\%) \quad (6)$$

$$\begin{aligned} \text{Percentage power} = \\ 100\% \times \text{antilog} (\text{dB} / 10) \end{aligned} \quad (7)$$

$$\begin{aligned} \text{Percentage voltage} = \\ 100\% \times \text{antilog} (\text{dB} / 20) \end{aligned} \quad (8)$$

Example: A power ratio of $20\% = 10 \log (20\% / 100\%) = 10 \log (0.2) = -7 \text{ dB}$

Example: A voltage ratio of $150\% = 20 \log (150\% / 100\%) = 20 \log (1.5) = 3.52 \text{ dB}$

Example: -1 dB represents a percentage power $= 100\% \times \text{antilog} (-1 / 10) = 79\%$

Example: 4 dB represents a percentage voltage $= 100\% \times \text{antilog} (4 / 20) = 158\%$

Computer Software and Calculators

Every version of the *Windows* operating system comes with a calculator program located in the Accessories program group and a number of free calculators are available on-line. Enter "online" and "calculator" into the search window of an Internet search engine for a list. If you can express your calculation as a mathematical expression, such as $\sin(45)$ or $10\log(17.5)$, it can be entered directly into the search window at www.google.com and the Google calculator will attempt to solve for the answer.

Microsoft *Excel* (and similar spreadsheet programs) also make excellent calculators. If you are unfamiliar with the use of spreadsheets, here are some online tutorials and help programs:

homepage.cs.uri.edu/tutorials/csc101/pc/excel97/excel.html

www.baycongroup.com/el0.htm

www.bcschools.net/staff/ExcelHelp.htm

For assistance in converting units of measurement, the Web site www.onlineconversion.com is very useful. The Google online unit converter can also be used by entering the required conversion, such as "12 gauss in tesla", into the Google search window.

Rectangular and Polar Coordinates

Graphs are drawings of what equations describe with symbols — they're both saying the same thing. Graphs are used to present a visual representation of what an equation expresses. The way in which mathematical quantities are positioned on the graph is called the *coordinate system*. *Coordinate* is another name for the numeric scales that divide the graph into regular units. The location of every point on the graph is described by a pair of *coordinates*.

The two most common coordinate systems used in radio are the *rectangular-coordinate* system shown in **Figure 1** (sometimes called *Cartesian coordinates*) and the *polar-coordinate* system shown in **Figure 2**.

The line that runs horizontally through the

Decibels In Your Head

Every time you increase the power by a factor of 2 times, you have a 3-dB increase of power. Every 4 times increase of power is a 6-dB increase of power. When you increase the power by 10 times, you have a power increase of 10 dB. You can also use these same values for a decrease in power. Cut the power in half for a 3-dB loss of power. Reduce the power to $\frac{1}{4}$ the original value for a 6-dB loss in power. If you reduce the power to $\frac{1}{10}$ of the original value you will have a 10-dB loss. The power-loss values are often written as negative values: -3 , -6 or -10 dB . The following tables show these common decibel values and ratios.

Common dB Values and Power Ratios

P_2/P_1	dB
0.1	-10
0.25	-6
0.5	-3
1	0
2	3
4	6
10	10

Common dB Values and Voltage Ratios

V_2/V_1	dB
0.1	-20
0.25	-12
0.5	-6
0.707	-3
1.0	0
1.414	3
2	6
4	12
10	20

You can also derive all the dB equivalents of integer ratios by adding or subtracting dB values. For example, to calculate the dB for a power ratio of $10/4$ (2.5), subtract the dB equivalents for 10 and 4: $10 - 6 = 4 \text{ dB}$. Similarly for a ratio of $10/2$ (5), $10 - 3 = 7 \text{ dB}$. The ratio of $5/4$ (1.25) is $7 - 6 = 1 \text{ dB}$ and so forth. The same trick can be used with the voltage ratios.

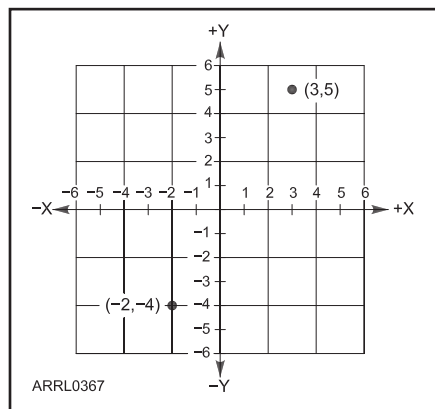


Figure 1 — Rectangular-coordinate graphs use a pair of axes at right angles to each other; each calibrate in numeric units. Any point on the resulting grid can be expressed in terms of its horizontal (X) and vertical (Y) values, called coordinates.

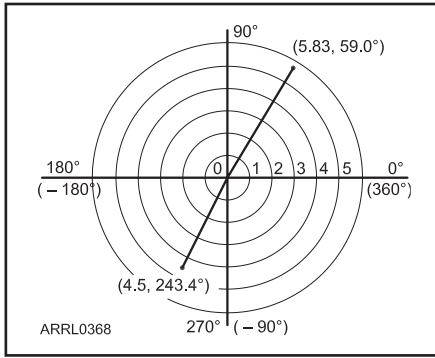


Figure 2 — Polar-coordinate graphs use a radius from the origin and an angle from the 0° axis to specify the location of a point. Thus, the location of any point can be specified in terms of a radius and an angle.

center of a rectangular coordinate graph is called the *X axis*. The line that runs vertically through the center of the graph is normally called the *Y axis*. Every point on a rectangular coordinate graph has two coordinates that identify its location, X and Y, also written as (X,Y). Every different pair of coordinate values describes a different point on the graph. The point at which the two axes cross — and where the numeric values on both axes are zero — is called the *origin*, written as (0,0).

In Figure 1, the point with coordinates (3,5) is located 3 units from the origin along the X axis and 5 units from the origin along the Y axis. Another point at (-2,-4) is found 2 units to the left of the origin along the X axis and 4 units below the origin along the Y axis. Do not confuse the X coordinate with the X representing reactance! Reactance is usually plotted on the Y-axis, creating occasional confusion.

In the polar-coordinate system, points on the graph are described by a pair of numeric values called *polar coordinates*. In this case, a length, or *radius*, is measured from the origin, and an *angle* is measured counterclockwise from the 0° line as shown in Figure 2. The symbol *r* is used for the radius and θ for the angle. A number in polar coordinates is written $r\angle\theta$. So the two points described in the last paragraph could also be written as $(5.83, \angle 59.0^\circ)$ and $(4.5, \angle 243.4^\circ)$. Unlike geographic maps, 0° is always to the *right* along the X-axis and 90° at the top along the Y-axis!

Angles are specified counterclockwise from the 0° axis. A negative value in front of an angle specifies an angle measured clockwise from the 0° axis. (Angles can also be given with reference to some arbitrary line, as well.) For example, -270° is equivalent to 90°, -90° is equivalent to 270°, 0° and

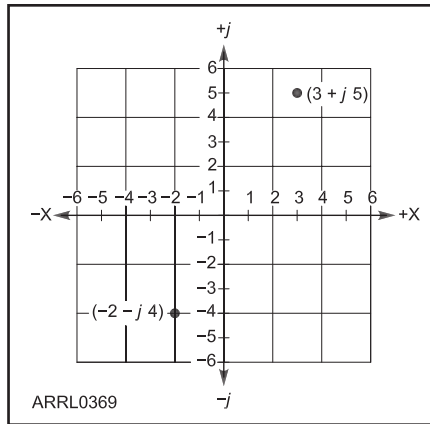


Figure 3 — The Y axis of a complex-coordinate graph represents the imaginary portion of complex numbers. This graph shows the same numbers as in Figure 1, graphed as complex numbers.

-360° are equivalent, and +180° and -180° are equivalent. With an angle measured clockwise from the 0° axis, the polar coordinates of the second point in Figure 2 would be $(4.5, \angle -116.6^\circ)$.

In some calculations the angle will be specified in *radians* (1 radian = $360 / 2\pi$ degrees = 57.3°), but you may assume that all angles are in degrees in this book. Radians are used when *angular frequency* (ω) is used instead of frequency (*f*). Angular frequency is measured in units of 2π radians/sec and so $\omega = 2\pi f$.

In electronics, it's common to use both the rectangular and polar-coordinate systems when dealing with impedance problems. The examples in the next few sections of this book should help you become familiar with these coordinate systems and the techniques for changing between them.

Complex Numbers

Mathematical equations that describe phases and angles of electrical quantities use the symbol *j* to represent the square root of minus one ($\sqrt{-1}$). (Mathematicians use *i* for the same purpose, but *i* is used to represent current in electronics.) The number *j* and any real number multiplied by *j* are called *imaginary numbers* because no real number is the square root of minus one. For example, $2j$, $0.1j$, $7j/4$, and $457.6j$ are all imaginary numbers. Imaginary numbers are used in electronics to represent reactance, such as $j13 \Omega$ of inductive reactance or $-j25 \Omega$ of capacitive reactance.

Real and imaginary numbers can be combined by using addition or subtraction. Adding a real number to an imaginary number

creates a hybrid called a *complex number*, such as $1 + j$ or $6 - 7j$. These numbers come in very handy in radio, describing impedances, relationships between voltage and current, and many other phenomena.

If the complex number is broken up into its real and imaginary parts, those two numbers can also be used as coordinates on a graph using *complex coordinates*. This is a special type of rectangular-coordinate graph that is also referred to as the *complex plane*. By convention, the X axis coordinates represent the real number portion of the complex number and Y axis coordinates the imaginary portion. For example, the complex number $6 - 7j$ would have the same location as the point (6,-7) on a rectangular-coordinate graph. Figure 3 shows the same points as Figure 1.

The number *j* has a number of interesting properties:

$$1/j = -j$$

$$j^2 = -1$$

$$j^3 = -j$$

$$j^4 = 1$$

Multiplication by *j* can also represent a phase shift or rotation of +90°. In polar coordinates $j = 1 \angle 90^\circ$.

WORKING WITH COMPLEX NUMBERS

Complex numbers representing electrical quantities can be expressed in either rectangular form ($a + jb$) or polar form ($r\angle\theta$) as described above. Adding complex numbers is easiest in rectangular form:

$$(a + jb) + (c + jd) = (a+c) + j(b+d) \quad (9)$$

Multiplying and dividing complex numbers is easiest in polar form:

$$a\angle\theta_1 \times b\angle\theta_2 = (a \times b) \angle (\theta_1 + \theta_2) \quad (10)$$

and

$$\frac{a\angle\theta_1}{b\angle\theta_2} = \left(\frac{a}{b}\right) \angle (\theta_1 - \theta_2) \quad (11)$$

Converting from one form to another is useful in some kinds of calculations. For example, to calculate the value of two complex impedances in parallel you use the formula

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

To calculate the numerator ($Z_1 Z_2$) you would write the impedances in polar form. To calculate the denominator ($Z_1 + Z_2$) you would write the impedances in rectangular

form. So you need to be able to convert back and forth from one form to the other. Here is the procedure:

To convert from rectangular ($a + jb$) to polar form ($r \angle \theta$):

$$r = \sqrt{a^2 + b^2} \quad \text{and} \quad \theta = \tan^{-1}(b/a) \quad (12)$$

To convert from polar to rectangular form:

$$a = r \cos \theta \quad \text{and} \quad b = r \sin \theta \quad (13)$$

Many calculators have polar-rectangular conversion functions built-in and they are worth learning how to use. Be sure that your calculator is set to the correct units for angles, radians or degrees.

Example: Convert $3 \angle 60^\circ$ to rectangular form:

$$a = 3 \cos 60^\circ = 3 (0.5) = 1.5$$

$$b = 3 \sin 60^\circ = 3 (0.866) = 2.6$$

$$3 \angle 60^\circ = 1.5 + j2.6$$

Example: Convert $0.8 + j0.6$ to polar form:

$$r = \sqrt{0.8^2 + 0.6^2} = 1$$

$$\theta = \tan^{-1}(0.6/0.8) = 36.8^\circ$$

$$0.8 + j0.6 = 1 \angle 36.8^\circ$$

Accuracy and Significant Figures

The calculations you have encountered in this chapter and will find throughout this handbook follow the rules for accuracy of calculations. Accuracy is represented by the number of *significant digits* in a number. That is, the number of digits that carry numeric value information beyond order of magnitude. For example, the numbers 0.123, 1.23,

12.3, 123, and 1230 all have three significant digits.

The result of a calculation can only be as accurate as its *least* accurate measurement or known value. This is important because it is rare for measurements to be more accurate than a few percent. This limits the number of useful significant digits to two or three. Here's another example; what is the current through a $12\text{-}\Omega$ resistor if 4.6 V is applied? Ohm's Law says I in amperes $= 4.6 / 12 = 0.3833333\ldots$ but because our most accurate numeric information only has two significant digits (12 and 4.6), strictly applying the significant digits calculation rule limits our answer to 0.38. One extra digit is often included, 0.383 in this case, to act as a guide in rounding off the answer.

Quite often a calculator is used, and the result of a calculation fills the numeric window. Just because the calculator shows 9 digits after the decimal point, this does not mean that is a more correct or even useful answer.